-1 حساب $I = \int_1^2 x\sqrt{2-x}\,dx$

نضع : $t = \sqrt{2-x}$ إذن $t^2 = 2-x$

أي $x = 2 - t^2$

ومنه : $dx = -2t\ dt$

من أجل $x = 1$ لدينا : $t = \sqrt{2-1} = 1$

ومن أجل $x = 2$ لدينا : $t = \sqrt{2-2} = 0$

إذن : $I = \int_1^0 (2-t^2)\,.\,t\,.\,(-2t\,dt) = 2\int_0^1 (2t^2 - t^4)\,dt$

$$= 2\left[\frac{2t^3}{3} - \frac{t^5}{5}\right]_0^1 = 2\left(\frac{2}{3} - \frac{1}{5}\right)$$

وبالتالي $I = \frac{14}{15}$

-2 أ – لنبين أن : $\sin^4 x \cos^4 x = \frac{1}{128}(\cos 8x - 4\cos 4x + 3)$

لدينا : $\sin^4 x \cos^4 x = \frac{1}{16}(2\sin x \cos x)^4$

$$= \frac{1}{16}(\sin 2x)^4$$

$$= \frac{1}{16}(\sin^2 2x)^2$$

$$= \frac{1}{16}\left(\frac{1-\cos 4x}{2}\right)^2$$

$$= \frac{1}{64}(1 - 2\cos 4x + \cos^2 4x)$$

$$= \frac{1}{64}\left(1 - 2\cos 4x + \frac{1+\cos 8x}{2}\right)$$

$$= \frac{1}{64}\left(\frac{\cos 8x - 4\cos 4x + 3}{2}\right)$$

إذن : $\sin^4 x \cos^4 x = \frac{1}{128}(\cos 8x - 4\cos 4x + 3)$

ب – حساب $J = \int_0^{\frac{\pi}{2}} \sin^4 x\,.\,\cos^4 x\,dx$

لدينا : $J = \frac{1}{128}\int_0^{\frac{\pi}{2}} (\cos 8x - 4\cos 4x + 3)\,dx$

$$= \frac{1}{128}\left[\frac{1}{8}\sin 8x - \frac{1}{16}\sin 4x + 3x\right]_0^{\frac{\pi}{2}} = \frac{1}{128}\left(\frac{3\pi}{2} - 0\right)$$

إذن : $J = \frac{3\pi}{256}$


تمتع
بالقراءة
مع الشامل