

$$I - J = \frac{\ln 2}{2} \quad \text{و} \quad I + J = \frac{\pi}{4} \quad : \text{ 1- لنبين أن}$$

$$I + J = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos x + \sin x} dx \quad \text{لدينا :}$$

$$= \int_0^{\frac{\pi}{4}} 1 \cdot dx = [x]_0^{\pi/4} = \frac{\pi}{4} - 0$$

$$I + J = \frac{\pi}{4} \quad \text{إذن :}$$

$$I - J = \int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\cos x + \sin x} dx \quad \text{ولدينا :}$$

Achamel

$$= \int_0^{\frac{\pi}{4}} \frac{(\cos x + \sin x)'}{\cos x + \sin x} dx = [\ln |\cos x + \sin x|]_0^{\frac{\pi}{4}}$$

$$= \ln \left( \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - \ln(1 + 0) = \ln \sqrt{2}$$

$$I - J = \frac{1}{2} \ln 2 \quad \text{إذن :}$$

### ب- استنتاج I و J

لدينا :  $I - J = \frac{1}{2} \ln 2$  و  $I + J = \frac{\pi}{4}$

إذن :

$$\begin{cases} I + J + I - J = \frac{\pi}{4} + \frac{1}{2} \ln 2 \\ I + J - I + J = \frac{\pi}{4} - \frac{1}{2} \ln 2 \end{cases}$$

أي :

$$\begin{cases} 2I = \frac{\pi}{4} + \frac{1}{2} \ln 2 \\ 2J = \frac{\pi}{4} - \frac{1}{2} \ln 2 \end{cases}$$

وبالتالي فإن :

$$I = \frac{\pi}{8} + \frac{1}{4} \ln 2$$

و

$$J = \frac{\pi}{8} - \frac{1}{4} \ln 2$$

2- حساب

$$K = \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{3}}} \frac{dx}{x^2 \sqrt{1-x^2}}$$

نضع :  $x = \frac{1}{t}$  أي  $t = \frac{1}{x}$

ومنه :  $dx = -\frac{1}{t^2} dt$

من أجل :  $t = \sqrt{2}$  لدينا  $x = \frac{1}{\sqrt{2}}$

ومن أجل :  $t = \sqrt{3}$  لدينا  $x = \frac{1}{\sqrt{3}}$

إذن :

$$K = \int_{\sqrt{2}}^{\sqrt{3}} \frac{-\frac{1}{t^2} dt}{\frac{1}{t^2} \sqrt{1 - \frac{1}{t^2}}}$$

$$\begin{aligned} &= \int_{\sqrt{3}}^{\sqrt{2}} \frac{dt}{\sqrt{\frac{t^2-1}{t^2}}} = \int_{\sqrt{3}}^{\sqrt{2}} \frac{dt}{\frac{1}{|t|}\sqrt{t^2-1}} \\ &= \int_{\sqrt{3}}^{\sqrt{2}} \frac{dt}{\frac{1}{t}\sqrt{t^2-1}} = \int_{\sqrt{3}}^{\sqrt{2}} \frac{t dt}{\sqrt{t^2-1}} = \int_{\sqrt{3}}^{\sqrt{2}} \frac{(t^2-1)'}{2\sqrt{t^2-1}} dt \\ &= \left[ \sqrt{t^2-1} \right]_{\sqrt{3}}^{\sqrt{2}} = \sqrt{2-1} - \sqrt{3-1} \\ &K = 1 - \sqrt{2} \end{aligned}$$

وبالتالي فإن :

Achamel