

1-1- إثبات المتساوية :

ليكن x عنصراً من $\mathbb{R} \setminus \{-2, 2\}$ لدينا :

$$\begin{aligned} -1 + \frac{1}{2-x} + \frac{1}{2+x} &= \frac{-(4-x^2) + 2+x + 2-x}{4-x^2} \\ &= \frac{x^2}{4-x^2} \end{aligned}$$

ب- حساب I :

لدينا :

$$\begin{aligned} I &= \int_{\frac{1}{2}}^1 \frac{x^2}{4-x^2} dx \\ &= \int_{\frac{1}{2}}^1 \left(-1 + \frac{1}{2-x} + \frac{1}{2+x} \right) dx \\ &= - \int_{\frac{1}{2}}^1 dx + \int_{\frac{1}{2}}^1 \frac{dx}{2-x} + \int_{\frac{1}{2}}^1 \frac{dx}{2+x} \end{aligned}$$

$$\begin{aligned}
 &= -\left[x \right]_{\frac{1}{2}}^1 + \left[\ln(2-x) \right]_{\frac{1}{2}}^1 + \left[\ln(2+x) \right]_{\frac{1}{2}}^1 \\
 &= -\left(1 - \frac{1}{2}\right) + \left(\ln 1 - \ln \frac{3}{2}\right) + \left(\ln 3 - \ln \frac{5}{2}\right) \\
 &= -\frac{1}{2} - (\ln 3 - \ln 2) + \ln 3 - (\ln 5 - \ln 2) \\
 &= -\frac{1}{2} - \ln 5 + 2 \ln 2
 \end{aligned}$$

2- حساب J :

$$\begin{cases} u(x) = x \\ v'(x) = \frac{-2x}{4-x^2} \end{cases} \text{ : ومنه } \quad \begin{cases} u'(x) = 1 \\ v(x) = \ln(4-x^2) \end{cases} \text{ : نضع}$$

$$J = \int_{\frac{1}{2}}^1 u'(x) \cdot v(x) \, dx \quad \text{ : ومنه}$$

$$= [u(x) \cdot v(x)]_{1/2}^1 - \int_{\frac{1}{2}}^1 u(x) \cdot v'(x) \, dx$$

$$= [x \ln(4-x^2)]_{1/2}^1 - \int_{\frac{1}{2}}^1 \frac{-2x^2}{4-x^2} \, dx$$

$$= \left(\ln 3 - \frac{1}{2} \ln \frac{15}{4} \right) + 2I$$

$$= \ln 3 - \frac{1}{2} \ln \frac{15}{4} + 2 \left(-\frac{1}{2} - \ln 5 + 2 \ln 2 \right)$$

$$= \ln 3 - \frac{1}{2} \left(\ln 5 + \ln 3 - \ln 4 \right) + 2 \left(-\frac{1}{2} - \ln 5 + 2 \ln 2 \right)$$

$$= -1 - \frac{1}{2} \ln 3 - \frac{5}{2} \ln 5 + 5 \ln 2$$

3- حساب K :

ليكن t عنصراً من $\left[3, \frac{15}{4} \right]$

$$u^2 = 4 - t \quad \text{ : نضع } \quad u = \sqrt{4-t} \quad \text{ : أي}$$

$$t = 4 - u^2 \quad \text{ : أي}$$

$$dt = -2u \, du \quad \text{ : ومنه}$$

$$u = 1 \quad \text{ : فإن } \quad t = 3 \quad \text{ : إذا كان}$$

$$u = \frac{1}{2} \quad \text{ : فإن } \quad t = \frac{15}{4} \quad \text{ : وإذا كان}$$

$$K = \int_1^{\frac{1}{2}} \frac{\ln(4-u^2)}{u} \cdot (-2u \, du) \quad \text{وبالتالي:}$$

$$= -2 \int_1^{\frac{1}{2}} \ln(4-u^2) \, du$$

$$= 2J$$
$$= -2 - \ln 3 - 5 \ln 5 + 10 \ln 2$$

Achamel



Achamel