1) نعلم أن
$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

و
$$(a - b)^3 = (a^3 - b^3) - 3\,ab\,(a - b)$$

$$\sin^3 x = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^3$$
$$= \frac{\left(e^{3ix} - e^{-3ix}\right) - 3e^{ix}\,.\,e^{ix}\left(e^{ix} - e^{-ix}\right)}{-8i}$$
$$= \frac{2i\sin 3x - 3\,.\,2i\sin x}{-8i}$$

إذن :
$$\boxed{a = \sin^3 x = \frac{3}{8}\sin x - \frac{1}{8}\sin 3x}$$

2) نعلم أن
$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

و
$$(a + b)^4 = a^4 + b^4 + 4ab\,(a^2 + b^b) + 6(ab)^2$$

إذن :
$$\cos^4 x = \left(\frac{e^{ix} + e^{-ix}}{2}\right)^4$$
$$= \frac{e^{i4x} + e^{-i4x} + 4\,e^{ix}\,.\,e^{-ix}\left(e^{i2x} + e^{-2ix}\right) + 6\left(e^{ix}\,.\,e^{-ix}\right)^2}{2^4}$$
$$= \frac{2\cos(4x) + 4.2\cos(2x) + 6}{16}$$

إذن :
$$\boxed{b = \cos^4 x = \frac{1}{8}\cos(4x) + \frac{1}{2}\cos(2x) + \frac{3}{8}}$$

3) لدينا $(a-b)^4 = (a^4 + b^4) - 4ab(a^2 + b^b) + 6(ab)^2$

إذن :
$$c = \sin^4 x = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^4$$

$$= \frac{(e^{i4x} + e^{-i4x}) - 4(e^{ix}.e^{-ix})\left(e^{i2x} + e^{-2ix}\right) + 6\left(e^{ix}.e^{-ix}\right)^2}{16}$$

$$= \frac{2\cos(4x) - 4.2\cos(2x) + 6}{16}$$

إذن :
$$\boxed{c = \sin^4 x = \frac{1}{8}\cos(4x) - \frac{1}{2}\cos(2x) + \frac{3}{8}}$$

4) لدينا

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$
$$= \left(a^5 + b^5\right) + 5ab\left(a^3 + b^3\right) + 10\left(a^2b^2\right)(a + b)$$

إذن :
$$d = \cos^5 x = \left(\frac{e^{ix} + e^{-ix}}{2}\right)^5$$

$$= \frac{\left(e^{i5x} + e^{-5ix}\right) + 5e^{ix}.e^{-ix}\left(e^{i3x} + e^{-3ix}\right) + 10e^{i2x}.e^{-2ix}\left(e^{ix} + e^{-ix}\right)}{32}$$

$$= \frac{2\cos(5x) + 5.2.\cos(3x) + 10.2\cos x}{32}$$

إذن :

$$\boxed{d = \cos^5 x = \frac{1}{16}\cos(5x) + \frac{5}{16}\cos(3x) + \frac{5}{8}\cos x}$$

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