

$$\forall \theta \in \mathbb{R} \quad 1 + e^{i\theta} = 2 \cos \frac{\theta}{2} \cdot e^{i \frac{\theta}{2}} \quad (1) \quad \text{لنبين أن}$$

\* الطريقة 1 :

$$\begin{aligned} 1 + e^{i\theta} &= 1 + \cos \theta + i \sin \theta \\ &= 2 \cos^2 \frac{\theta}{2} + 2 i \sin \frac{\theta}{2} \cos \frac{\theta}{2} \\ &= 2 \cos \frac{\theta}{2} \left[ \cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right] \end{aligned}$$

$$\boxed{1 + e^{i\theta} = 2 \cos \frac{\theta}{2} \cdot e^{i \frac{\theta}{2}}} \quad \text{إذن}$$

\* الطريقة 2 :

$$\begin{aligned} 1 + e^{i\theta} &= e^{i\left(\frac{\theta}{2} - \frac{\theta}{2}\right)} + e^{i\left(\frac{\theta}{2} + \frac{\theta}{2}\right)} \\ &= e^{i \frac{\theta}{2}} \left( e^{-i \frac{\theta}{2}} + e^{i \frac{\theta}{2}} \right) \\ &= e^{i \frac{\theta}{2}} \cdot 2 \cos \frac{\theta}{2} \end{aligned}$$

$$z_1 = 1 + e^{i \frac{\pi}{8}} \quad * (2)$$

$$= 2 \cos \frac{\pi}{16} \cdot e^{i \frac{\pi}{16}}$$

$$\boxed{z_1 = \left[ 2 \cos \frac{\pi}{16}, \frac{\pi}{16} \right]} \quad \text{فإن } \cos \frac{\pi}{16} > 0 \quad \text{بما أن}$$

$$\begin{aligned} z_2 &= 1 + e^{i \frac{5\pi}{3}} \\ &= 2 \cos \frac{5\pi}{6} \cdot e^{i \frac{5\pi}{6}} \end{aligned}$$

$$z_2 = \left[ -2 \cos \frac{5\pi}{6}, \frac{5\pi}{6} + \pi \right] \quad \text{فإن } \cos \frac{5\pi}{6} < 0 \quad \text{بما أن}$$

$$\boxed{z_2 = \left[ \sqrt{3}, \frac{11\pi}{6} \right]} \quad \text{أي}$$

$$\begin{aligned} z_3 &= \frac{2 + \sqrt{3} + i}{2} & * \\ &= 1 + \frac{\sqrt{3}}{2} + \frac{1}{2} i \\ &= 1 + e^{i \frac{\pi}{6}} \\ &= 2 \cos \frac{\pi}{12} \cdot e^{i \frac{\pi}{12}} \end{aligned}$$

$$z_3 = \left[ 2 \cos \frac{\pi}{12}, \frac{\pi}{12} \right] \quad \text{إذن}$$

$$\begin{aligned} z_4 &= \frac{2 + \sqrt{2} - i \sqrt{2}}{2} & * \\ &= 1 + \frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \\ &= 1 + e^{-i \frac{\pi}{4}} \\ &= 2 \cos \left( -\frac{\pi}{8} \right) e^{-i \frac{\pi}{8}} \end{aligned}$$

$$z_4 = \left[ 2 \cos \frac{\pi}{8}, -\frac{\pi}{8} \right] \quad \text{إذن}$$

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