

$$\alpha \in \mathbb{R} \text{ حيث } (E) : z^2 - 2z \sin \alpha + 2(1 + \cos \alpha) = 0$$

$$\begin{aligned} \Delta &= (2 \sin \alpha)^2 - 4 \cdot 2(1 + \cos \alpha) & (1) \\ &= 4 \sin^2 \alpha - 4 \cdot 2(1 + \cos \alpha) \\ &= 4(1 - \cos^2 \alpha - 2 - 2 \cos \alpha) \\ &= -4(\cos^2 \alpha + 2 \cos \alpha + 1) \\ &= [2i(1 + \cos \alpha)]^2 \end{aligned}$$

$$\begin{aligned} z_1 &= \frac{2 \sin \alpha + 2i(1 + \cos \alpha)}{2} \\ &= \sin \alpha + i(1 + \cos \alpha) \end{aligned}$$

$$\begin{aligned} z_2 &= \frac{2 \sin \alpha - 2i(1 + \cos \alpha)}{2} \\ &= \sin \alpha - i(1 + \cos \alpha) \end{aligned}$$

إذن

$$S = \{ \sin \alpha + i(1 + \cos \alpha); \sin \alpha - i(1 + \cos \alpha) \}$$

(2) لدينا :

$$\begin{aligned} z_1 &= \sin \alpha + i(1 + \cos \alpha) \\ &= 2 \cos \left(\frac{\alpha}{2} \right) \sin \left(\frac{\alpha}{2} \right) + i 2 \cos^2 \left(\frac{\alpha}{2} \right) \end{aligned}$$

$$\begin{aligned} &= 2 \cos \left(\frac{\alpha}{2} \right) \left[\sin \left(\frac{\alpha}{2} \right) + i \cos \left(\frac{\alpha}{2} \right) \right] \\ &= 2 \cos \left(\frac{\alpha}{2} \right) \left[\cos \left(\frac{\pi - \alpha}{2} \right) + i \left(\frac{\pi - \alpha}{2} \right) \right] \end{aligned}$$

ولدينا $z_2 = \overline{z_1}$

إذن

* إذا كان $\cos \left(\frac{\alpha}{2} \right) > 0$ فإن $|z_1| = |z_2| = 2 \cos \left(\frac{\alpha}{2} \right)$

و $\text{Arg}(z_2) \equiv \frac{\alpha - \pi}{2} [2\pi]$ و $\text{Arg}(z_1) \equiv \frac{\alpha - \pi}{2} [2\pi]$

* إذا كان $\cos \left(\frac{\alpha}{2} \right) < 0$ فإن $|z_1| = |z_2| = -2 \cos \left(\frac{\alpha}{2} \right)$

و $\text{Arg}(z_2) \equiv \frac{\alpha - 3\pi}{2} [2\pi]$ و $\text{Arg}(z_1) \equiv \frac{3\pi - \alpha}{2} [2\pi]$

* إذا كان $\cos \left(\frac{\alpha}{2} \right) = 0$ فإن $z_1 = z_2 = 0$

ومنه فإن $|z_1| = |z_2| = 0$ والعمدة غير محدد.

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