$(E) : z^2 - 2z\sin\alpha + 2(1+\cos\alpha) = 0$ حيث $\alpha \in IR$.

1) $\Delta = (2\sin\alpha)^2 - 4.2(1+\cos\alpha)$
$= 4\sin^2\alpha - 4.2(1+\cos\alpha)$
$= 4(1-\cos^2\alpha - 2 - 2\cos\alpha)$
$= -4(\cos^2\alpha + 2\cos\alpha + 1)$
$= [2i(1+\cos\alpha)]^2$

$$z_1 = \frac{2\sin\alpha + 2i(1+\cos\alpha)}{2} = \sin\alpha + i(1+\cos\alpha)$$

$$z_2 = \frac{2\sin\alpha - 2i(1+\cos\alpha)}{2} = \sin\alpha - i(1+\cos\alpha)$$

إذن

$$S = \{\sin\alpha + i(1+\cos\alpha) ; \sin\alpha - i(1+\cos\alpha)\}$$

2) لدينا :

$$z_1 = \sin\alpha + i(1+\cos\alpha) = 2\cos\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right) + i\,2\cos^2\left(\frac{\alpha}{2}\right)$$

$$= 2\cos\left(\frac{\alpha}{2}\right)\left[\sin\left(\frac{\alpha}{2}\right) + i\cos\left(\frac{\alpha}{2}\right)\right]$$

$$= 2\cos\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\pi-\alpha}{2}\right) + i\left(\frac{\pi-\alpha}{2}\right)\right]$$

ولدينا $z_2 = \overline{z_1}$

إذن

* إذا كان $\cos\left(\frac{\alpha}{2}\right) > 0$ فإن $|z_1| = |z_2| = 2\cos\left(\frac{\alpha}{2}\right)$

و $Arg(z_1) \equiv \frac{\alpha - \pi}{2}\,[2\pi]$ و $Arg(z_2) \equiv \frac{\alpha-\pi}{2}\,[2\pi]$

* إذا كان $\cos\left(\frac{\alpha}{2}\right) < 0$ فإن $|z_1| = |z_2| = -2\cos\left(\frac{\alpha}{2}\right)$

و $Arg(z_1) \equiv \frac{3\pi - \alpha}{2}\,[2\pi]$ و $Arg(z_2) \equiv \frac{\alpha - 3\pi}{2}\,[2\pi]$

* إذا كان $\cos\left(\frac{\alpha}{2}\right) = 0$ فإن $z_1 = z_2 = 0$

ومنه فإن $|z_1| = |z_2| = 0$ والعمدة غير محدد.

إعداد الأستاذ : سعيد الهاني