* نضع : $\begin{cases} u(x) = \ln x \\ v'(x) = 1 \end{cases}$ و $\begin{cases} u'(x) = \frac{1}{x} \\ v(x) = x \end{cases}$

$$I_1 = \int_1^e \ln x \, dx$$

$$= [x \ln x]_1^e - \int_1^e x . \frac{1}{x} \, dx$$

$$= [x \ln x]_1^e - [x]_1^e$$

$$= 1$$

* نضع : $\begin{cases} u(x) = x \\ v'(x) = e^x \end{cases}$ و $\begin{cases} u'(x) = 1 \\ v(x) = e^x \end{cases}$

إذن :

$$I_2 = \int_1^{\ln 2} x e^x \, dx$$

$$= [x e^x]_1^{\ln 2} - \int_1^{\ln 2} e^x \, dx$$

$$= [x e^x - e^x]_1^{\ln 2}$$

$$= 2 \ln 2 - 2$$

* نضع : $\begin{cases} u(x) = x + 1 \\ v'(x) = \cos x \end{cases}$ و $\begin{cases} u'(x) = 1 \\ v(x) = \sin x \end{cases}$

إذن :

$$I_3 = \int_0^{\frac{\pi}{2}} (x + 1) \cos x \, dx$$

$$= [(x + 1) \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx$$

$$= [(x + 1)\sin x + \cos x]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

* نضع : $\begin{cases} u(x) = \ln x \\ v'(x) = x \end{cases}$ و $\begin{cases} u'(x) = \frac{1}{x} \\ v(x) = \frac{1}{2}x^2 \end{cases}$

إذن :

$$I_4 = \int_1^e x \ln x \, dx$$

$$= \left[\frac{1}{2}x^2 \ln x\right]_1^e - \int_1^e \frac{1}{2}x \, dx$$

$$= \frac{1}{2}e^2 - \left[\frac{1}{4}x^2\right]_1^e$$

$$= \frac{1}{4}(e^2 + 1)$$

* نضع : $\begin{cases} u(x) = \ln x \\ v'(x) = \sqrt{x} \end{cases}$ و $\begin{cases} u'(x) = \frac{1}{x} \\ v(x) = \frac{2}{3}x\sqrt{x} \end{cases}$

إذن :

$$I_5 = \int_1^{e^2} \sqrt{x} \ln x \, dx$$

$$= \left[\frac{2}{3}x\sqrt{x} \ln x\right]_1^{e^2} - \int_1^{e^2} \frac{2}{3}\sqrt{x} \, dx$$

$$= \left[\frac{2}{3}x\sqrt{x} \ln x - \frac{4}{9}x\sqrt{x}\right]_1^{e^2}$$

$$= \frac{2e^3 + 4}{9}$$

* نضع : $\begin{cases} u(x) = \ln x \\ v'(x) = x^2 + x + 1 \end{cases}$ و $\begin{cases} u'(x) = \frac{1}{x} \\ v(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x \end{cases}$

إذن :

$$I_6 = \int_1^e (x^2 + x + 1) \ln x \, dx$$

$$= \left[\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right)\ln x\right]_1^e - \int_1^e \left(\frac{1}{3}x^2 + \frac{1}{2}x + 1\right)dx$$

$$= \left[\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right)\ln x\right]_1^e - \left[\frac{1}{9}x^3 + \frac{1}{4}x^2 + x\right]_1^e$$

$$= \frac{2}{9}e^3 + \frac{1}{4}e^2 + \frac{49}{36}$$