

(1) نضع $u(x) = x^2 + 1$ إذن $u'(x) = 2x$

$$\begin{aligned} \int_0^1 \frac{x}{(x^2 + 1)^2} dx &= \frac{1}{2} \int_0^1 \frac{u'(x)}{(u(x))^2} dx \quad \text{ومنہ :} \\ &= \frac{1}{2} \left[-\frac{1}{u(x)} \right]_0^1 \\ &= -\frac{1}{2} \left[\frac{1}{x^2 + 1} \right]_0^1 \\ &= \frac{1}{4} \end{aligned}$$

(2) نضع $u(x) = \cos x$ إذن $u'(x) = -\sin x$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \tan x dx &= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx \quad \text{ومنہ :} \\ &= - \int_0^{\frac{\pi}{4}} \frac{u'(x)}{u(x)} dx \\ &= - [\ln |u(x)|]_0^{\frac{\pi}{4}} \\ &= - [\ln (\cos x)]_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} \ln 2 \end{aligned}$$

$$I = \int_0^{\frac{\pi}{4}} \frac{x}{\cos^2 x} dx \quad \text{(b) نعتبر التكامل}$$

$$\begin{cases} u'(x) = 1 \\ v(x) = \tan x \end{cases} \text{ إذن } \begin{cases} u(x) = x \\ v'(x) = \frac{1}{\cos^2 x} \end{cases} \text{ نضع} :$$

التكامل I يصبح :

$$\begin{aligned} I &= \int_0^{\frac{\pi}{4}} \frac{x}{\cos^2 x} dx \\ &= \int_0^{\frac{\pi}{4}} u(x) v'(x) dx \\ &= [u(x) v(x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} u'(x) v(x) dx \\ &= [x \tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x dx \\ &= \frac{\pi}{4} - \frac{1}{2} \ln 2 \end{aligned}$$