

1) أ- ليكن  $x$  عددا حقيقيا مخالفا للعددين 0 و 1 -.

لدينا :

$$\begin{aligned} \frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1} &= \frac{x+1 - x(x+1) + x^2(x+1) - x^3}{x^3(x+1)} \\ &= \frac{x+1 - x^2 - x + x^3 + x^2 - x^3}{x^3(x+1)} \\ &= \frac{1}{x^3(x+1)} \end{aligned}$$

ب- من خلال السؤال السابق لدينا :

$$\begin{aligned} \int_1^2 \frac{1}{x^3(x+1)} dx &= \int_1^2 \left( \frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1} \right) dx \\ &= \int_1^2 \frac{1}{x^3} dx - \int_1^2 \frac{1}{x^2} dx + \int_1^2 \frac{1}{x} dx - \int_1^2 \frac{1}{x+1} dx \\ &= \left[ -\frac{1}{2} \frac{1}{x^2} \right]_1^2 + \left[ \frac{1}{x} \right]_1^2 + [\ln x]_1^2 - [\ln(x+1)]_1^2 \\ &= -\frac{1}{8} + 2 \ln 2 - \ln 3 \\ &= -\frac{1}{8} + \ln \frac{4}{3} \end{aligned}$$

(2) نحسب التكامل :  $\int_1^4 \frac{1}{t^2(1+\sqrt{t})} dt$

نضع  $x = \sqrt{t}$  . إذن  $dx = \frac{1}{2\sqrt{t}} dt$  أي  $dt = 2x dx$

إذا كان  $t = 4$  فإن  $x = 2$  وإذا كان  $t = 1$  فإن  $x = 1$ .

$$\begin{aligned} \int_1^4 \frac{dt}{t^2(1+\sqrt{t})} &= \int_1^2 \frac{2x dx}{x^4(1+x)} \quad \text{ومنه :} \\ &= 2 \int_1^2 \frac{dx}{x^3(1+x)} \\ &= 2 \left( -\frac{1}{8} + \ln \frac{4}{3} \right) \\ &= -\frac{1}{4} + 2 \ln \frac{4}{3} \end{aligned}$$



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